

## Variational Principles In Classical Mechanics

*This textbook provides lecture materials of a comprehensive course in Classical Mechanics developed by the author over many years with input from students and colleagues alike. The richly illustrated book covers all major aspects of mechanics starting from the traditional Newtonian perspective, over Lagrangian mechanics, variational principles and Hamiltonian mechanics, rigid-body, and continuum mechanics, all the way to deterministic chaos and point-particle mechanics in special relativity. Derivation steps are worked out in detail, illustrated by examples, with ample explanations.Developed by a classroom practitioner, the book provides a comprehensive overview of classical mechanics with judicious material selections that can be covered in a one-semester course thus streamlining the instructor's task of choosing materials for their course. The usefulness for instructors notwithstanding, the primary aim of the book is to help students in their understanding, with detailed derivations and explanations, and provide focused guidance for their studies by repeatedly emphasizing how various topics are tied together by common physics principles.*

*Mechanics is one of the oldest and at the same time newest disciplines, in the sense that there are methods and principles developed first in mechanics but now widely used in almost all branches of physics: electrodynamics, quantum mechanics, classical and quantum field theory, special and general theory of relativity, etc. More than that, there are some formalisms like Lagrangian and Hamiltonian approaches, which represent the key stone for the development of the above-mentioned disciplines. During the last 20-25 years, classical mechanics has undergone an important revival associated with the progress in non-linear dynamics, applications of Noether's theorem and the extension of variational principles in various interdisciplinary sciences (for instance, magnetofluid dynamics). Thus, there ought to exist a book concerned with the applied analytical formalism, first developed in the frame of theoretical mechanics, which has proved to be one of the most efficient tools of investigation in the entire arena of science. The present book is an outcome of the authors' teaching experience over many years in different countries and for different students studying diverse fields of physics. The book is intended for students at the level of undergraduate and graduate studies in physics, engineering, astronomy, applied mathematics and for researchers working in related subjects. We hope that the original presentation and the distribution of the topics, the various applications in many branches of physics and the set of more than 100 proposed problems, shall make this book a comprehensive and useful tool for students and researchers. The present book is an outcome of the authors' teaching experience over many years in different countries and for different students studying diverse fields of physics. The book is intended for students at the level of undergraduate and graduate studies in physics, engineering, astronomy, applied mathematics and for researchers working in related subjects. We hope that the original presentation and the distribution of the topics, the various applications in many branches of physics and the set of more than 100 proposed problems, shall make this book a comprehensive and useful tool for students and researchers.*

*This is the fifth edition of a well-established textbook. It is intended to provide a thorough coverage of the fundamental principles and techniques of classical mechanics, an old subject that is at the base of all of physics, but in which there has also in recent years been rapid development. The book is aimed at undergraduate students of physics and applied mathematics. It emphasizes the basic principles, and aims to progress rapidly to the point of being able to handle physically and mathematically interesting problems, without getting bogged down in excessive formalism. Lagrangian methods are introduced at a relatively early stage, to get students to appreciate their use in simple contexts. Later chapters use Lagrangian and Hamiltonian methods extensively, but in a way that aims to be accessible to undergraduates, while including modern developments at the appropriate level of detail. The subject has been developed considerably recently while retaining a truly central role for all students of physics and applied mathematics. This edition retains all the main features of the fourth edition, including the two chapters on geometry of dynamical systems and on order and chaos, and the new appendices on conics and on dynamical systems near a critical point. The material has been somewhat expanded, in particular to contrast continuous and discrete behaviours. A further appendix has been added on routes to chaos (period-doubling) and related discrete maps. The new edition has also been revised to give more emphasis to specific examples worked out in detail. Classical Mechanics is written for undergraduate students of physics or applied mathematics. It assumes some basic prior knowledge of the fundamental concepts and reasonable familiarity with elementary differential and integral calculus. Contents: Linear MotionEnergy and Angular MomentumCentral Conservative ForcesRotating FramesPotential TheoryThe Two-Body ProblemMany-Body SystemsRigid BodiesLagrangian MechanicsSmall Oscillations and Normal ModesHamiltonian MechanicsDynamical Systems and Their GeometryOrder and Chaos in Hamiltonian SystemsAppendices:VectorsConicsPhase Plane Analysis Near Critical PointsDiscrete Dynamical Systems — Maps Readership: Undergraduates in physics and applied mathematics.*

*The main purpose of the book is to acquaint mathematicians, physicists and engineers with classical mechanics as a whole, in both its traditional and its contemporary aspects. As such, it describes the fundamental principles, problems, and methods of classical mechanics, with the emphasis firmly laid on the working apparatus, rather than the physical foundations or applications. Chapters cover the n-body problem, symmetry groups of mechanical systems and the corresponding conservation laws, the problem of the integrability of the equations of motion, the theory of oscillations and perturbation theory.*

*\* Offers a rigorous mathematical treatment of mechanics as a text or reference \* Revisits beautiful classical material, including gyroscopes, precessions, spinning tops, effects of rotation of the Earth on gravity motions, and variational principles \* Employs mathematics not only as a "unifying" language, but also to exemplify its role as a catalyst behind new concepts and discoveries*

*Recent years have seen a growing trend to derive models of macroscopic phenomena encountered in the fields of engineering, physics, chemistry, ecology, self-organisation theory and econophysics from various variational or extremum principles. Through the link between the integral extremum of a functional and the local extremum of a function (explicit, for example, in the Pontryagin's maximum principle variational and extremum principles are mutually related. Thus it makes sense to consider them within a common context. The main goal of Variational and Extremum Principles in Macroscopic Systems is to collect various mathematical formulations and examples of physical reasoning that involve both basic theoretical aspects and applications of variational and extremum approaches to systems of the macroscopic world. The first part of the book is focused on the theory, whereas the second focuses on applications. The unifying variational approach is used to derive the balance or conservation equations, phenomenological equations linking fluxes and forces, equations of change for processes with coupled transfer of energy and substance, and optimal conditions for energy management. A unique multidisciplinary synthesis of variational and extremum principles in theory and application A comprehensive review of current and past achievements in variational formulations for macroscopic processes Uses Lagrangian and Hamiltonian formalisms as a basis for the exposition of novel approaches to transfer and conversion of thermal, solar and chemical energy*

*Two dramatically different philosophical approaches to classical mechanics were developed during the 17th - 18th centuries. Newton developed his vectorial formulation that uses time-dependent differential equations of motion to relate vector observables like force and rate of change of momentum. Euler, Lagrange, Hamilton, and Jacobi, developed powerful alternative variational formulations based on the assumption that nature follows the principle of least action. These powerful variational formulations have become the preeminent philosophical approach used in modern science, was well as having applications to other fields such as economics and engineering.This book introduces variational principles, and illustrates the intellectual beauty, the remarkable power, and the broad scope, of applying variational principles to classical mechanics. A brief review of Newtonian mechanics compares and contrasts the relative merits of the intuitive Newtonian vectorial formulation, with the more powerful analytical variational formulations. Applications presented cover a wide variety of topics, as well as extensions to accommodate relativistic mechanics, and quantum theory.*

*Given a conservative dynamical system of classical physics, how does one find a variational principle for it? Is there a canonical recipe for such a principle? The case of particle mechanics was settled by Lagrange in 1788; this text treats continuous systems. Recipes devised are algebraic in nature, and this book develops all the mathematical tools found necessary after the minute examination of the adiabatic fluid dynamics in the introduction. These tools include: Lagrangian and Hamiltonian formalisms, Legendre transforms, dual spaces of Lie algebras and associated 2-cocycles; and linearized and Z2-graded versions of all of these. The following typical physical systems, together with their Hamiltonian structures, are discussed: Classical Magnetohydro-dynamics with its Hall deformation; Multifluid Plasma; Superfluid He-4 (both irrotational and rotating) and 3He-A; Quantum fluids; Yang-Mills MHD; Spinning fluids; Spin Glass; Extended YM Plasma; A Lattice Gas. Detailed motivations, easy-to-follow arguments, open problems, and over 300 exercises help the reader. Request Inspection Copy*

***[Foundations of Classical Mechanics](#)***

***[Perfect Form](#)***

***[Classical Mechanics: Lecture Notes](#)***

***[A Student's Guide to Lagrangians and Hamiltonians](#)***

***[An Introduction to the Principle of Least Action](#)***

***[Variational Principles, Methods, and Applications in Elementary Physics](#)***

***[Tensors, Differential Forms, and Variational Principles](#)***

***[Classical Mechanics with Calculus of Variations and Optimal Control](#)***

***[Variational Methods in Theoretical Mechanics](#)***

The book reviews the two features of the variational approach: its use as a universal tool to describe physical phenomena and as a source for qualitative and quantitative methods of studying particular problems. Berdichevsky's work differs from other books on the subject in focusing mostly on the physical origin of variational principles as well as establishing their interrelations. For example, the Gibbs principles appear as a consequence of the Einstein formula for thermodynamic fluctuations rather than as the first principles of the theory of thermodynamic equilibrium. Mathematical issues are considered as long as they shed light on the physical outcomes and/or provide a useful technique for the direct study of variational problems. In addition, a thorough account of variational principles discovered in various branches of continuum mechanics is given. This book, the second volume, describes how the variational approach can be applied to constructing models of continuum media, such as the theory of elastic plates; shells and beams; shallow water theory; heterogeneous mixtures; granular materials; and turbulence. It goes on to apply the variational approach to asymptotical analysis of problems with small parameters, such as the derivation of the theory of elastic plates, shells and beams from three-dimensional elasticity theory; and the basics of homogenization theory. A theory of stochastic variational problems is considered in detail too, along with applications to the homogenization of continua with random microstructures.

This textbook introduces readers to the detailed and methodical resolution of classical and more recent problems in analytical mechanics. This valuable learning tool includes worked examples and 40 exercises with step-by-step solutions, carefully chosen for their importance in classical, celestial and quantum mechanics. The collection comprises six chapters, offering essential exercises on: (1) Lagrange Equations; (2) Hamilton Equations; (3) the First Integral and Variational Principle; (4) Canonical Transformations; (5) Hamilton – Jacobi Equations; and (6) Phase Integral and Angular Frequencies Each chapter begins with a brief theoretical review before presenting the clearly solved exercises. The last two chapters are of particular interest, because of the importance and flexibility of the Hamilton-Jacobi method in solving many mechanical problems in classical mechanics, as well as quantum and celestial mechanics. Above all, the book provides students and teachers alike with detailed, point-by-point and step-by-step solutions of exercises in Lagrangian and Hamiltonian mechanics, which are central to most problems in classical physics, astronomy, celestial mechanics and quantum physics.

CHOICE Recommended Title, June 2019 Brought together in one focused and exclusive treatment, this book provides an elementary introduction to the important role and use of the least action principle and the resulting Lagrange's equations in the analysis of the laws that govern the universe. It is an ideal complimentary resource to accompany undergraduate courses and textbooks on classical mechanics. Features: Uses mathematics accessible to beginners Brings together the Principle of Least Action, Lagrange's equations, and variational principles in mechanics in one cohesive text Written in a clear and easy-to-understand manner

This is a translation of A.I. Lurie's classical Russian textbook on analytical mechanics. It offers a consummate exposition of the subject of analytical mechanics through a deep analysis of its most fundamental concepts. The book has served as a desk text for at least two generations of researchers working in those fields where the Soviet Union accomplished the greatest technological breakthrough of the 20th century - a race into space. Those and other related fields continue to be intensively explored since then, and the book clearly demonstrates how the fundamental concepts of mechanics work in the context of up-to-date engineering problems.

This book constructs the mathematical apparatus of classical mechanics from the beginning, examining basic problems in dynamics like the theory of oscillations and the Hamiltonian formalism. The author emphasizes geometrical considerations and includes phase spaces and flows, vector fields, and Lie groups. Discussion includes qualitative methods of the theory of dynamical systems and of asymptotic methods like averaging and adiabatic invariance.

What does the path taken by a ray of light share with the trajectory of a thrown baseball and the curve of a wheat stalk bending in the breeze? Each is the subject of a different study yet all are optimal shapes: light rays minimize travel time while a thrown baseball minimizes action. All natural curves and shapes, and many artificial ones, manifest such "perfect form" because physical principles can be expressed as a statement requiring some important physical quantity to be mathematically maximum, minimum, or stationary. Perfect Form introduces the basic "variational" principles of classical physics (least time, least potential energy, least action, and Hamilton's principle), develops the mathematical language most suited to their application (the calculus of variations), and presents applications from the physics usually encountered in introductory course sequences. The text gradually unfolds the physics and mathematics. While other treatments postulate Hamilton's principle and deduce all results from it, Perfect Form begins with the most plausible and restricted variational principles and develops more powerful ones through generalization. One selection of text and problems even constitutes a non-calculus of variations introduction to variational methods, while the mathematics more generally employed extends only to solving simple ordinary differential equations. Perfect Form is designed to supplement existing classical mechanics texts and to present variational principles and methods to students who approach the subject for the first time.

This is a rare book on a rare topic: it is about 'action' and the Principle of Least Action. A surprisingly well-kept secret, these ideas are at the heart of physical science and engineering. Physics is well known as being concerned with grand conservatory principles (e.g. the conservation of energy) but equally important is the optimization principle (such as getting somewhere in the shortest time or with the least resistance). The book explains: why an optimization principle underlies physics, what action is, what `the Hamiltonian' is, and how new insights into energy, space, and time arise. It assumes some background in the physical sciences, at the level of undergraduate science, but it is not a textbook. The requisite derivations and worked examples are given but may be skim-read if desired. The author draws from Cornelius Lanczos's book "The Variational Principles of Mechanics" (1949 and 1970). Lanczos was a brilliant mathematician and educator, but his book was for a postgraduate audience. The present book is no mere copy with the difficult bits left out - it is original, and a popularization. It aims to explain ideas rather than achieve technical competence, and to show how Least Action leads into the whole of physics.

Gregory's Classical Mechanics is a major new textbook for undergraduates in mathematics and physics. It is a thorough, self-contained and highly readable account of a subject many students find difficult. The author's clear and systematic style promotes a good understanding of the subject: each concept is motivated and illustrated by worked examples, while problem sets provide plenty of practice for understanding and technique. Computer assisted problems, some suitable for projects, are also included. The book is structured to make learning the subject easy; there is a natural progression from core topics to more advanced ones and hard topics are treated with particular care. A theme of the book is the importance of conservation principles. These appear first in vectorial mechanics where they are proved and applied to problem solving. They reappear in analytical mechanics, where they are shown to be related to symmetries of the Lagrangian, culminating in Noether's theorem.

***[ANALYTICAL DYNAMICS](#)***

***[II. Applications](#)***

***[Lagrangian and Hamiltonian Analytical Mechanics: Forty Exercises Resolved and Explained](#)***

***[CLASSICAL MECHANICS](#)***

***[Universe Dynamics](#)***

***[The Variational Principles of Mechanics](#)***

***[Lagrangian and Hamiltonian Dynamics](#)***

***[Variational Principles in Classical Mechanics - Second Edition](#)***

***[An Intuitive Introduction](#)***

This book brings together the essential ideas and methods behind applications of variational theory in theoretical physics and chemistry. The emphasis is on understanding physical and computational applications of variational methodology rather than on rigorous mathematical formalism. The text begins with an historical survey of familiar variational principles in classical mechanics and optimization theory, then proceeds to develop the variational principles and formalism behind current computational methodology for bound and continuum quantum states of interacting electrons in atoms, molecules, and condensed matter. It covers multiple-scattering theory, including a

detailed presentation of contemporary methodology for electron-impact rotational and vibrational excitation of molecules. The book ends with an introduction to the variational theory of relativistic fields. Ideal for graduate students and researchers in any field that uses variational methodology, this book is particularly suitable as a backup reference for lecture courses in mathematical methods in physics and theoretical chemistry.

This is an intuitively motivated presentation of many topics in classical mechanics and related areas of control theory and calculus of variations. All topics throughout the book are treated with zero tolerance for unrevealing definitions and for proofs which leave the reader in the dark. Some areas of particular interest are: an extremely short derivation of the ellipticity of planetary orbits; a statement and an explanation of the "tennis racket paradox"; a heuristic explanation (and a rigorous treatment) of the gyroscopic effect; a revealing equivalence between the dynamics of a particle and statics of a spring; a short geometrical explanation of Pontryagin's Maximum Principle, and more. In the last chapter, aimed at more advanced readers, the Hamiltonian and the momentum are compared to forces in a certain static problem. This gives a palpable physical meaning to some seemingly abstract concepts and theorems. With minimal prerequisites consisting of basic calculus and basic undergraduate physics, this book is suitable for courses from an undergraduate to a beginning graduate level, and for a mixed audience of mathematics, physics and engineering students. Much of the enjoyment of the subject lies in solving almost 200 problems in this book.

An introductory textbook exploring the subject of Lagrangian and Hamiltonian dynamics, with a relaxed and self-contained setting. Lagrangian and Hamiltonian dynamics is the continuation of Newton's classical physics into new formalisms, each highlighting novel aspects of mechanics that gradually build in complexity to form the basis for almost all of theoretical physics. Lagrangian and Hamiltonian dynamics also acts as a gateway to more abstract concepts routed in differential geometry and field theories and can be used to introduce these subject areas to newcomers. Journeying in a self-contained manner from the very basics, through the fundamentals and onwards to the cutting edge of the subject, along the way the reader is supported by all the necessary background mathematics, fully worked examples, thoughtful and vibrant illustrations as well as an informal narrative and numerous fresh, modern and inter-disciplinary applications. The book contains some unusual topics for a classical mechanics textbook. Most notable examples include the 'classical wavefunction', Koopman-von Neumann theory, classical density functional theories, the 'vakonomic' variational principle for non-holonomic constraints, the Gibbs-Appell equations, classical path integrals, Nambu brackets and the full framing of mechanics in the language of differential geometry.

This textbook provides an introduction to classical mechanics at a level intermediate between the typical undergraduate and advanced graduate level. This text describes the background and tools for use in the fields of modern physics, such as quantum mechanics, astrophysics, particle physics, and relativity. Students who have had basic undergraduate classical mechanics or who have a good understanding of the mathematical methods of physics will benefit from this book.

A comprehensive guide to using energy principles and variational methods for solving problems in solid mechanics This book provides a systematic, highly practical introduction to the use of energy principles, traditional variational methods, and the finite element method for the solution of engineering problems involving bars, beams, torsion, plane elasticity, trusses, and plates. It begins with a review of the basic equations of mechanics, the concepts of work and energy, and key topics from variational calculus. It presents virtual work and energy principles, energy methods of solid and structural mechanics, Hamilton's principle for dynamical systems, and classical variational methods of approximation. And it takes a more unified approach than that found in most solid mechanics books, to introduce the finite element method. Featuring more than 200 illustrations and tables, this Third Edition has been extensively reorganized and contains much new material, including a new chapter devoted to the latest developments in functionally graded beams and plates. Offers clear and easy-to-follow descriptions of the concepts of work, energy, energy principles and variational methods Covers energy principles of solid and structural mechanics, traditional variational methods, the least-squares variational method, and the finite element, along with applications for each Provides an abundance of examples, in a problem-solving format, with descriptions of applications for equations derived in obtaining solutions to engineering structures Features end-of-the-chapter problems for course assignments, a Companion Website with a Solutions Manual, Instructor's Manual, figures, and more Energy Principles and Variational Methods in Applied Mechanics, Third Edition is both a superb text/reference for engineering students in aerospace, civil, mechanical, and applied mechanics, and a valuable working resource for engineers in design and analysis in the aircraft, automobile, civil engineering, and shipbuilding industries.

This book provides a concise description of the current status of a fascinating scientific problem — the inverse variational problem in classical mechanics. The essence of this problem is as follows: one is given a set of equations of motion describing a certain classical mechanical system, and the question to be answered is: Do these equations of motion correspond to some Lagrange function as its Euler–Lagrange equations? In general, not for every system of equations of motion does a Lagrange function exist; it can, however, happen that one may modify the given equations of motion in such a way that they yield the same set of solutions as the original ones and they correspond already to a Lagrange function. Moreover, there can even be infinitely many such Lagrange functions, the relations among which are not trivial. The book deals with this scope of problems. No advanced mathematical methods, such as, contemporary differential geometry, are used. The intention is to meet the standard educational level of a broad group of physicists and mathematicians. The book is well suited for use as lecture notes in a university course for physicists. Contents:Constants of MotionTheorem of HenneauxInstructive Example of DouglasConstruction of the Most General Autonomous One-Particle Lagrange Function in (3+1) Space–Time Dimensions Giving Rise to Rotationally Covariant Euler–Lagrange EquationsEvaluation of the Function GijConstruction of the Most General Two-Particle Lagrange Function in (1+1) Space–Time Dimensions Giving Rise to Euler–Lagrange Equations Covariant Under Galilei TransformationGalilei Forminvariance of the Euler–Lagrange Equations for Two Particles in (1+1) Space–Time Dimensions Readership: Graduate students of theoretical physics and mathematics, as well as theoretical physicists doing research in classical and quantum mechanics. Keywords:Variational Calculus;Classical Mechanics;Lagrange Function;Galilei Group;Euler-Lagrange;Constants of Motion;Linear Partial Differential Equations;Inverse Variational Problem;Poincare Lemma;Helmholtz Conditions;Theorem of Henneaux;Hamilton Formalism;S-Equivalence of Lagrange Functions;Weakly Symmetrical Lagrange Functions;Douglas' Method;Poisson Bracket;Lagrange Bracket

DIVHistorical, theoretical survey with many insights, much hard-to-find material. Hamilton's principle, Hamilton-Jacobi equation, etc. /div

A concise treatment of variational techniques, focussing on Lagrangian and Hamiltonian systems, ideal for physics, engineering and mathematics students.

[The Least Action Principle and Lagrange's Equations](#)

[Analytical Mechanics](#)

[The Lazy Universe](#)

[The Variational Principles of Dynamics](#)

[The Inverse Variational Problem in Classical Mechanics](#)

[Theory and Mathematical Modeling](#)

[Variational Principles of Classical Mechanics](#)

[A History of the Calculus of Variations from the 17th through the 19th Century](#)

[Variational and Extremum Principles in Macroscopic Systems](#)

As a limit theory of quantum mechanics, classical dynamics comprises a large variety of phenomena, from computable (integrable) to chaotic (mixing) behavior. This book presents the KAM (Kolmogorov-Arnold-Moser) theory and asymptotic completeness in classical scattering. Including a wealth of fascinating examples in physics, it offers not only an excellent selection of basic topics, but also an introduction to a number of current areas of research in the field of classical mechanics. Thanks to the didactic structure and concise appendices, the presentation is self-contained and requires only knowledge of the basic courses in mathematics.

The book addresses the needs of graduate and senior undergraduate students in mathematics and physics, and of researchers interested in approaching classical mechanics from a modern point of view.

The new edition of a classic text that concentrates on developing general methods for studying the behavior of classical systems, with extensive use of computation.

Philosophic, less formalistic approach to analytical mechanics offers model of clear, scholarly exposition at graduate level with coverage of basics, calculus of variations, principle of virtual work, equations of motion, more.

Intended as a text for postgraduate students of mathematics, this compact and well-organized book offers insights into the principles of classical mechanics and, in particular, deals with the problems of dynamical systems. Divided into seven chapters, the text begins with a discussion on some elementary results of statics and dynamics. It then goes on to analyze at length the Hamiltonian formulation along with the Poisson bracket, the variational principle (taking Euler's equation of calculus of variation as the base), and different forms of the variational principle. Finally, the text explains the integral invariants, canonical transformations, and the Hamilton–Jacobi theory. KEY FEATURES □ A fairly large number of worked-out examples are interspersed throughout the text to illustrate the application of the concepts to the problems discussed. □ Miscellaneous Exercises are given at the end of the book to drill the students in self-study. □ The text entirely covers UGC model curriculum for M.Sc. (Mathematics).

"Two dramatically different philosophical approaches to classical mechanics were developed during the 17th - 18th centuries. Newton developed his vectorial formulation that uses time-dependent differential equations of motion to relate vector observables like force and rate of change of momentum. Euler, Lagrange, Hamilton, and Jacobi, developed powerful alternative variational formulations based on the assumption that nature follows the principle of least action. These powerful variational formulations have become the preeminent philosophical approach used in modern science, as well as having applications to other fields such as economics and engineering. This book introduces variational principles, and illustrates the intellectual beauty, the remarkable power, and the broad scope, of applying variational principles to classical mechanics. A brief review of Newtonian mechanics compares and contrasts the relative merits of the intuitive Newtonian vectorial formulation, with the more powerful analytical variational formulations. Applications presented cover a wide variety of topics, as well as extensions to accommodate relativistic mechanics, and quantum theory."--Open Textbook Library.

The calculus of variations is a subject whose beginning can be precisely dated. It might be said to begin at the moment that Euler coined the name calculus of variations but this is, of course, not the true moment of inception of the subject. It would not have been unreasonable if I had gone back to the set of isoperimetric problems considered by Greek mathematicians such as Zenodorus (c. 200 B. C. ) and preserved by Pappus (c. 300 A. D. ). I have not done this since these problems were solved by geometric means. Instead I have arbitrarily chosen to begin with Fermat's elegant principle of least time. He used this principle in 1662 to show how a light ray was refracted at the interface between two optical media of different densities. This analysis of Fermat seems to me especially appropriate as a starting point: He used the methods of the calculus to minimize the time of passage of a light ray through the two media, and his method was adapted by John Bernoulli to solve the brachistochrone problem. There have been several other histories of the subject, but they are now hopelessly archaic. One by Robert Woodhouse appeared in 1810 and another by Isaac Todhunter in 1861.

This is a textbook written for use in a graduate-level course for students of mechanics and engineering science. It is designed to cover the essential features of modern variational methods and to demonstrate how a number of basic mathematical concepts can be used to produce a unified theory of variational mechanics. As prerequisite to using this text, we assume that the student is equipped with an introductory course in functional analysis at a level roughly equal to that covered, for example, in Kolmogorov and Fomin (Functional Analysis, Vol. I, Graylock, Rochester, 1957) and possibly a graduate-level course in continuum mechanics. Numerous references to supplementary material are listed throughout the book. We are indebted to Professor Jim Douglas of the University of Chicago, who read an earlier version of the manuscript and whose detailed suggestions were extremely helpful in preparing the final draft. He also gratefully acknowledge that much of our own research work on variational theory was supported by the U.S. Air Force Office of Scientific Research. He are indebted to Mr. Ming-Goei Sheu for help in proofreading. Finally, we wish to express thanks to Mrs. Marilyn Gude for her excellent and pains taking job of typing the manuscript. J. T. ODEN J. N. REDDY Table of Contents PREFACE 1. INTRODUCTION 1.1 The Role of Variational Theory in Mechanics. 1.1.2 Some Historical Comments ..... 2 1.3 Plan of Study ..... 5 7 2. MATHEMATICAL FOUNDATIONS OF CLASSICAL VARIATIONAL THEORY 7 2.1 Introduction . . . . .

There are about 500 books on variational principles. They are concerned mostly with the mathematical aspects of the topic. The major goal of this book is to discuss the physical origin of the variational principles and the intrinsic interrelations between them. For example, the Gibbs principles appear not as the first principles of the theory of thermodynamic equilibrium but as a consequence of the Einstein formula for thermodynamic quantities. The mathematical issues are considered as long as they shed light on the physical outcomes and/or provide a useful technique for direct study of variational problems.

The book is a completely rewritten version of the author's monograph Variational Principles of Continuum Mechanics which appeared in Russian in 1983. I have been postponing the English translation because I wished to include the variational principles of irreversible processes in the new edition. Reaching an understanding of this subject took longer than I expected. In its final form, this book covers all aspects of the story. The part concerned with irreversible processes is tiny, but it determines the accents put on all the results presented. The other new issues included in the book are: entropy of microstructure, variational principles of vortex line dynamics, variational principles and integration in functional spaces, some stochastic variational problems, variational principle for probability densities of local fields in composites with random structure, variational theory of turbulence; these topics have not been covered previously in monographic literature.

[2nd Edition](#)

[Variational Principles in Physics](#)

[Variational Principles of Continuum Mechanics](#)

[Mathematical Aspects of Classical and Celestial Mechanics](#)

[Variational Principles in Classical Mechanics](#)

[An Intensive Course](#)

[Classical Mechanics](#)

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[Mathematical Physics: Classical Mechanics](#)

Two dramatically different philosophical approaches to classical mechanics were proposed during the 17th - 18th centuries. Newton developed his vectorial formulation that uses time-dependent differential equations of motion to relate vector observables like force and rate of change of momentum. Euler, Lagrange, Hamilton, and Jacobi, developed powerful alternative variational formulations based on the assumption that nature follows the principle of least action. These variational formulations now play a pivotal role in science and engineering. This book introduces variational principles and their application to classical mechanics. The relative merits of the intuitive Newtonian vectorial formulation, and the more powerful variational formulations are compared. Applications to a wide variety of topics illustrate the intellectual beauty, remarkable power, and broad scope provided by use of variational principles in physics. The second edition adds discussion of the use of variational principles applied to the following topics: (1) Systems subject to initial boundary conditions (2) The hierarchy of related formulations based on action, Lagrangian, Hamiltonian, and equations of motion, to systems that involve symmetries. (3) Non-conservative systems. (4) Variable-mass systems. (5) The General Theory of Relativity. Douglas Cline is a Professor of Physics in the Department of Physics and Astronomy, University of Rochester, Rochester, New York.

Incisive, self-contained account of tensor analysis and the calculus of exterior differential forms, interaction between the concept of invariance and the calculus of variations. Emphasis is on analytical techniques. Includes problems.

The books Fractional Calculus with Applications in Mechanics: Vibrations and Diffusion Processes and Fractional Calculus with Applications in Mechanics: Wave Propagation, Impact and Variational Principles contain various applications of fractional calculus to the fields of classical mechanics. Namely, the books study problems in fields such as viscoelasticity of fractional order, lateral vibrations of a rod of fractional order type, lateral vibrations of a rod positioned on fractional order viscoelastic foundations, diffusion-wave phenomena, heat conduction, wave propagation, forced oscillations of a body attached to a rod, impact and variational principles of a Hamiltonian type. The books will be useful for graduate students in mechanics and applied mathematics, as well as for researchers in these fields. Part 1 of this book presents an introduction to fractional calculus. Chapter 1 briefly gives definitions and notions that are needed later in the book and Chapter 2 presents definitions and some of the properties of fractional integrals and derivatives. Part 2 is the central part of the book. Chapter 3 presents the analysis of waves in fractional viscoelastic materials in infinite and finite spatial domains. In Chapter 4, the problem of oscillations of a translatory moving rigid body, attached to a heavy, or light viscoelastic rod of fractional order type, is studied in detail. In Chapter 5, the authors analyze a specific engineering problem of the impact of a viscoelastic rod against a rigid wall. Finally, in Chapter 6, some results for the optimization of a functional containing fractional derivatives of constant and variable order are presented.

Variational principles have proven to be surprisingly fertile. For example, Fermat used variational methods to demonstrate that light follows the fastest route from one point to another, an idea which came to be a cornerstone of geometrical optics. This book explains variational principles and charts their use throughout modern physics. It examines the analytical mechanics of Lagrange and Hamilton, the basic tools of any physicist. The book also offers simple but rich first impressions of Einstein's General Relativity, Feynman's Quantum Mechanics, and more that reveal amazing interconnections between various fields of physics.

[I. Fundamentals](#)

[Fractional Calculus with Applications in Mechanics](#)  
[Mathematical Methods of Classical Mechanics](#)  
[Energy Principles and Variational Methods in Applied Mechanics](#)  
[Variational Principles and Methods in Theoretical Physics and Chemistry](#)  
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[Wave Propagation, Impact and Variational Principles](#)  
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